



## Reducing the orders of mixed model (ARMA) before and after the wavelet de-noising with application

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### Abstract

In this paper, the estimated linear model of Box-Jenkins such as ARMA(p,q) has been compared from time series observations ,before and after wavelet shrinkage filtering (used to solve the problem of contamination (or noise) if it found in the observations) and then reducing the order of the estimated model from filtered observations (with preserving the accuracy and suitability of the estimated models) and re-compared with the estimated linear model of original observations , depending on some statistical criteria , including the Root Mean Square Error (RMSE) , the Mean Absolute Error (MAE) , and the Akaike's Information Criterion (AIC) ,through taking practical application of time series (the housing permits) by using statistical programs (Statgraphics, NCS and MATLAB). The results showed the efficiency of wavelet shrinkage filters in solving the noise problem and obtaining the efficient estimated models, and specifically the wavelet shrinkage filter (dmey) with Soft threshold which estimated it's level using the Fixed Form method of filtered observations, and the possibility of obtaining linear models of the filtered observations with lower orders and higher efficiency compared with the corresponding estimated model of original observations.

**Key words and phrases:** time series, Box-Jenkins, wavelet shrinkage, Filters, Fixed Form.

### 1. Introduction

The objectives of time series analysis are diverse, depending on the background of applications. Statisticians usually view a time series as a realization from a stochastic process. A fundamental task is to unveil the probability law that governs the observed time series. With such a probability law, we can understand the underlying dynamics, forecast future events, and control future events via intervention. Those are the three main objectives of time series analysis. The process of reducing the noise or removed before analysis the time series is very important in order to obtain more accurate and reliable results when building models. The wavelet shrinkage technique consists of wavelets with threshold is a strong mathematical approach to remove most of the noise while retaining the maximum amount of energy data that represents the real observation.

Wavelet transform have been used in many fields. It has been observed that the accuracy of the forecasting can be improved through using wavelet transforms.[1] used Saudi stock index to show that wavelet transform is better

Than the other forecasting technique in predicting the de-noising of the financial time series, and this is done after making a comparison with several forecasting models. Alwadi S., Mohd, Alkhahazaleh M.H. and Samsul Addul Karim,in 2011[2] used the wavelet transform to decompose the return Amman stock market in to a set of better behaved series data and explained that after making a comparison framework, the wavelet ARIMA model is better than the ARIMA model of the original data and gives more accurate results and also gives data with more stable in variance , mean and no outliers.

The basic idea of this paper is to see the method of the wavelet transforms in general and wavelet shrinkage in time series analysis in particular. Moreover, the study try to show the advantages in model estimation and forecasting through de-noising the series using wavelet shrinkage and try to get the ability of lowering the order of the estimated model using wavelet shrinkage. Housing permits data [3] was used for analysis (Data are summarized in appendix).

Two different methods were considered here, as showed in Figure1. In the first method, the housing permits data is modeled using Box-Jenkins method. Then some forecasting criteria were computed. In second method the technique of wavelet shrinkage was used through filtering the time series data using a set of wavelet filters. The de-noised series are modeled, as in the first method, and then the forecasting criteria were computed again. The forecasting criteria were evaluated and compared to those of the first method.

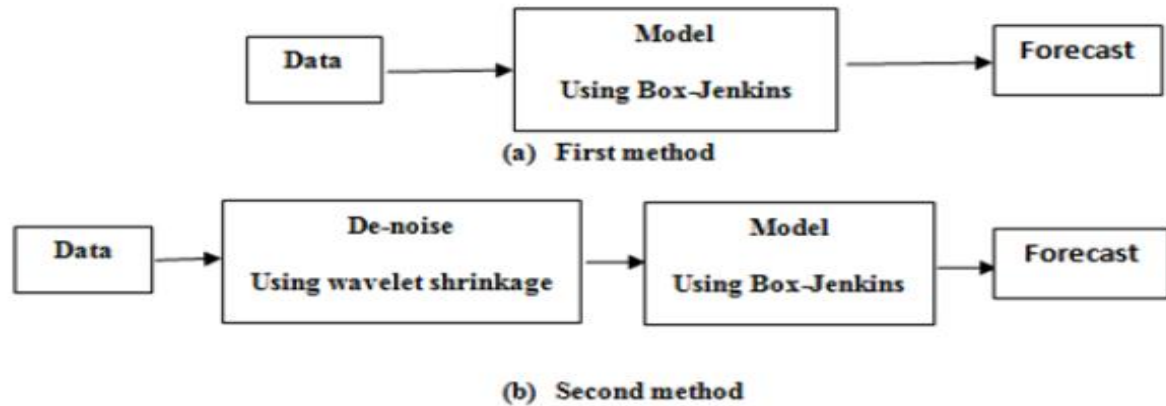


Figure 1. Modeling Methods

The remainder of this paper is structured as follows. In section 2 we review the simple brief introduction to wavelet transform. Section 3 gives time series de-noising using wavelet shrinkage .Section 4 deals with main results before and after reducing the order of the model using wavelet shrinkage and gives discussion. And finally in section 5 conclusions and 6 recommendations are presented.

## 2. The Discrete Wavelet Transform (DWT)

The mathematical transformations are applied to a signal to get additional information that is not found in the time domain representation of that signal. First we consider the Fourier transform, which decomposes signals in to sum of periodic bases of finite length and can transform the frequency domain and vice versa. It is defined as follows [4]:

Given a vector of a signal  $X$  consisting of  $2^j$  observation where  $j$  is integer. The Discrete Wavelet Transform (DWT) of  $X$  is

$$W = wX \quad (1)$$

$W$  is an orthonormal  $N * N$  matrix associated with the orthonormal wavelet basis chosen. and  $W$  is an  $n * 1$  vector comprising both discrete scaling and wavelet coefficients. The vector of wavelet coefficients can by organized into  $j + 1$  vectors.

$$W = [W1, W2, \dots, Wj_0, Vj_0]^T$$

Where  $Wj$  is a length  $(N_j = N/2^j)$  vector of wavelet coefficients (Details) associated with changes on a scale of length  $\lambda_j = 2^{j-1}$  and  $Vj_0$  is a length  $N_{j_0} = N/2^j$  vector of scaling coefficients (approximation or smoothing) associated with average on a scale of length  $\lambda_{j_0} = 2^{j_0}$ .

After each DWT, the approximation coefficients are divided into bands using the same filter as before, with the result that the details are appended with the details of the latest decomposition, at each level, the signal can be reconstructed of the de-noise signal by the inverse transform.



$$X = W w^T = \sum_{j=1}^{j_0} W_j^T W_j + V_{j_0}^T V_{j_0} \quad (2)$$

This can be illustrated wavelets ( $w$ ) used in this research follows:

#### A-Haar Wavelet:

Wavelet Haar is one of the simplest types of wavelets used for the purposes of analysis, has come into being through the thesis presented by the world (Alfred Haar) in the period (1909-1910) and because of its simplicity and ease they are considered the best option when wanting to learn and study of wavelets. and known that there two function play a key role in the wavelet analysis, wavelet function  $\Psi$ , or the so-called mother wavelet function and Scaling function  $\Phi$  for the wavelet function can be expressed by the formula [5]:

$$\Psi(u) = \begin{cases} 1, & 0 \leq u < \frac{1}{2} \\ -1, & \frac{1}{2} \leq u \leq 1 \\ 0 & \text{otherwise} \end{cases} \quad (3)$$

The function of the scale is expressed as follows: -

$$\phi(u) = \begin{cases} 1, & 0 \leq u \leq 1 \\ 0, & \text{Otherwise} \end{cases} \quad (4)$$

#### B-Discrete Meyer wavelet (dmey):

Wavelet Discrete (Meyer) or the so-called (dmey) is one of the world's productions (Yves Meyer), who is credited with the development of methods of analysis of wavelets. Mother wavelet function and Scale function in the frequency domain, and can be expressed through the following formula [4]:

$$\Psi(u) = \begin{cases} (2\pi)^{-1/2} e^{-iu/2} \sin\left(\frac{\pi}{2} v \left(\frac{3}{2\pi}|u| - 1\right)\right) & \text{if } \frac{2\pi}{3} \leq |u| \leq \frac{4\pi}{3} \\ (2\pi)^{-1/2} e^{iu/2} \cos\left(\frac{\pi}{2} v \left(\frac{3}{4\pi}|u| - 1\right)\right) & \text{if } \frac{4\pi}{3} \leq |u| \leq \frac{8\pi}{3} \\ 0 & \text{if } |u| \notin \left[\frac{2\pi}{3}, \frac{8\pi}{3}\right] \end{cases} \quad (5)$$

Where:

$$v(a) = a^4(35 - 84a + 70a^2 - 20a^3) \quad a \in [0,1]$$

The function of the scale is expressed as follows: -

$$\phi(u) = \begin{cases} (2\pi)^{-1/2} & \text{if } |u| \leq \frac{2\pi}{3} \\ (2\pi)^{-1/2} \cos\left(\frac{\pi}{2} v \left(\frac{3}{2\pi}|u| - 1\right)\right) & \text{if } \frac{2\pi}{3} \leq |u| \leq \frac{4\pi}{3} \\ 0 & \text{if } |u| > \frac{4\pi}{3} \end{cases} \quad (6)$$

#### C-Daubechies Wavelets

These wavelets named after the researcher (Ingrid Daubechies), which is considered the leading researcher on the subject of wavelet, has invented the so-called orthonormal wavelets with Compact Support in (1988), making discrete wavelet analysis applicable. And writes this family filters (DN) or (dbL1), where (D) and (db) is the name of the researcher (Daubechies) The (N) is the length of the candidate or his rank [8], while (L1) is the number of vanishing moments for wavelet function.

In general, the (dbN) represent a family band with rank (N) (note that Haar wavelet is a member of this family are the same because db1 its Haar wavelet). Wavelet properties can be incorporated as follows [9]:

1. Support of Wavelet (dbN) is on the period  $[0, 2N-1]$ .
2. Wavelet (dbN) to her (N) of vanishing moments.
3. The form of the functions of the Wavelet (dbN) far from symmetry.
4. Increasingly Regularity or Smoothness of the Wavelet (dbN) with increasing length of the candidate or his rank, any  $(0.2075N)$ , which represents the regularity index.
5. Orthogonal wavelet analysis.

#### D-Coiflets wavelets

Created the researcher (Daubechies) these wavelets based on a request by the researcher (Coifman) in the spring of 1989 and attributed to him, where the researcher introduced the idea of getting on the moments vanishing or transient filters scrolling the low and filters scrolling trade together for both functions ( $\Phi$  and  $\Psi$  instead be moments vanishing limited on ( $\Psi$ ) alone). These are called wavelet (Coif N) where (Coif) is an abbreviation (Coifman) while the (N) representing the rank of the candidate, and there is a relationship between the rank of the candidate with the length of which (the length of the candidate =  $6N$ ) and the number of moments vanishing of the function wavelet ( $\Psi$ ) is ( $L = 2N$ ), while the number of vanishing moments of the function of scale ( $\phi$ ) is ( $L_1 = 2N - 1$ ). Wavelet properties can be incorporated as follows [6]: -

1. Compact Support.
2. Orthogonal.
3. near symmetry.

#### 3. Time series analysis using wavelet shrinkage

Statistical wavelet methods provide a powerful tool to recover the original signal from noisy observations under general assumptions; the generic methodology is called wavelet shrinkage or wavelet thresholding. The underlying model for the noisy series can be expressed as follows:

$$X(t) = s(t) + n(t) \quad (7)$$

Where  $n(t)$  represents the independent and identically distributed random variables with zero mean and unite variance. The objective is to suppress the noise part of the series  $x(t)$  and recover the clean part  $s(t)$ . The procedure can be summarized as three main steps [11]:

- 1-The discrete wavelet transform DWT is computed from the data.
- 2-Significantly large coefficients in the DWT are kept, others are shrinked.
- 3-The inverse DWT is applied to the shrunken set of coefficients.

The three steps above can be summarizing in figure 2:

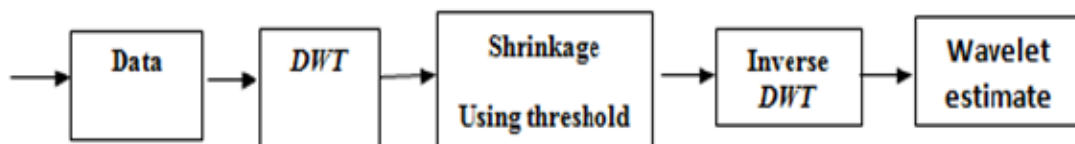


Figure 2. Wavelet shrinkage

For each level, we will have a threshold. The Fixed Form threshold (i.e.; Universal threshold) technique is considered and is given by the following formula:

$$\delta^{(FT)} = \hat{\sigma}(MAD) \sqrt{2 \log(N)} \quad (8)$$

Where (N) is the number of wavelet coefficients in specified level,  $\hat{\sigma}(MAD)$  is the estimate of the noise standard deviation and can be obtained by applying a median absolute deviation (MAD) estimator to the  $(N/2)$  wavelet coefficients at the first level of decomposition,

incorporating a scale factor equal to (0.6745) [5]. After estimating the threshold of a specified level, wavelet coefficient of that level are either hard or soft threshold. Hard thresholding represents the keep or kill (i.e.; wavelet coefficient is less than the threshold it would be put to zero otherwise it stays without change). The soft thresholding shrinks all nonzero coefficients towards zero, which gives a smooth de-noising .The soft threshold de-noising function formula can expressed as follows:

$$d_{J,k} = \begin{cases} d_{J,k}, & |d_{J,k}| \geq \delta, \\ 0, & |d_{J,k}| < \delta \end{cases} \quad (9)$$

Where  $d_{J,k}$  denotes the coefficient of transformation and  $\delta$  is the threshold, soft thresholding has smaller variance than hard thresholding, therefore here we only consider soft thresholding for modeling and forecasting the yields data of the housing permits in this paper.

#### 4. Main results and discussion

In this section, an application was considered so as to show first the ability of wavelet shrinkage to reduce the noise from original data, and second to get a model with a lower order and higher efficiency compared with the original. Figure 3 shows six levels multiresolution wavelet analysis using Haar wavelet for the yields from a housing permits process for 80 consecutive observations (Data are summarized in appendix), where  $s$  is the signal and it is the sum of its approximation and of its fine details , approximation at level 6 and  $d_6, d_5, d_4, d_3, d_2, d_1$  are the details at level 6, 5,4,3,2 and 1.

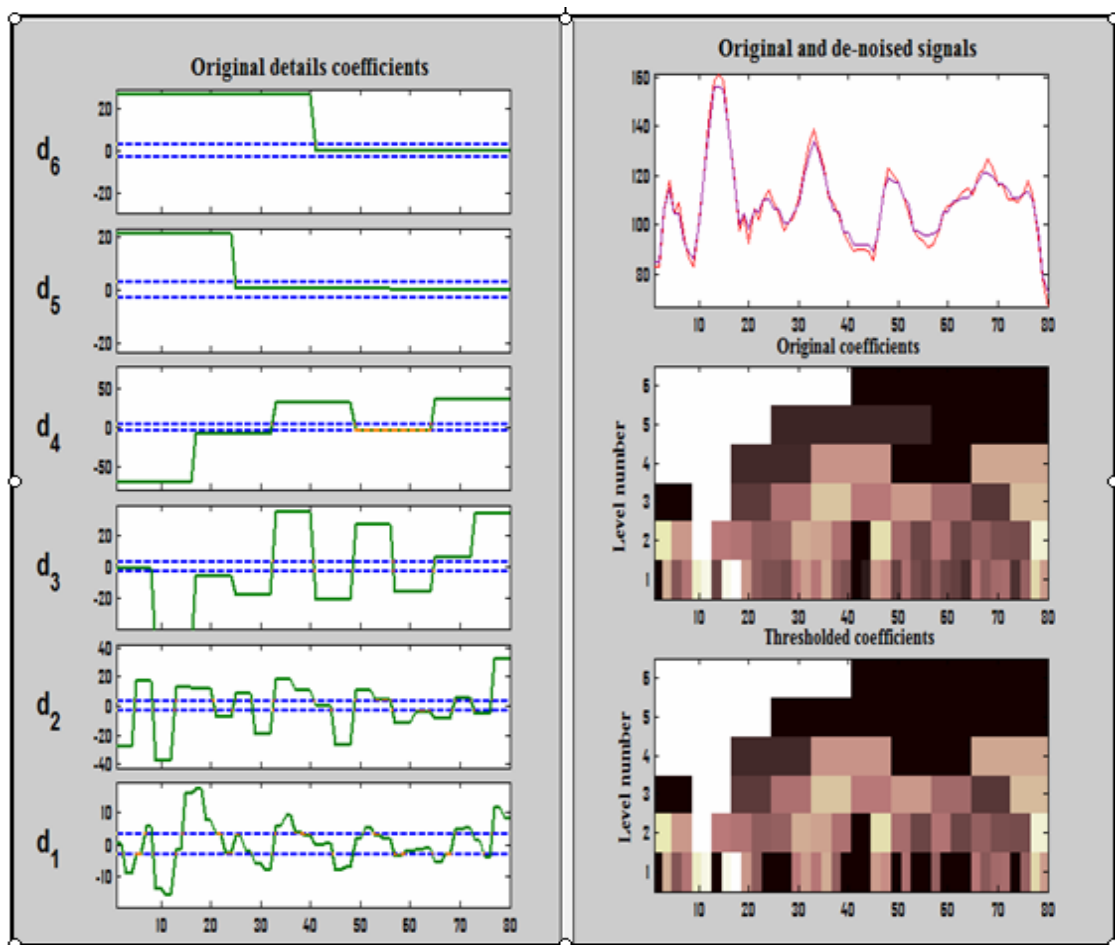


Figure 3. Six-multiresolution wavelet analysis using Haar wavelet for the yields from housing permits process



Now, we will use the two different methods mentioned in the introduction as follows: Method (1): The yields of housing permits process was modeled as ARMA(3, 2), the parameters (p = 3; q = 2) were selected after careful modeling and fitting (Statgraphics software was used for modeling). The performance measures used in the analysis are Root Mean Square Error (RMSE), Mean Absolute Error (MAE) and Akaike's Information Criterion(AIC) are computed as the following [8,12,13]:

$$RMSE = \sqrt{\frac{\sum_{t=1}^n a_t^2}{n - k}} \quad \dots(10)$$

$$MAE = \frac{\sum_{t=1}^n |a_t|}{n} \quad \dots(11)$$

$$AIC = Ln\sigma_a^2 + \frac{2k}{n} \quad \dots(12)$$

Where  $a_t$  represents the difference between actual and forecasted value, (k) is the number of estimated parameters of the model and (n) is the sample size. Method (2): The yields from a housing permits data noise model were considered. The data was de-noised using wavelet shrinkage technique mentioned in section (3) using MATLAB software, version 2008 [9] with four different wavelet families. It is important here to say that after many experimental trials with many wavelets families (Data are summarized in appendix), it was found that these wavelets better than others in terms of de-noising, housing permits data and they are:

- 1- Haar wavelet with six multiresolution levels.
- 2- Discrete Meyer (dmey) wavelet with six multiresolution levels.
- 3- Daubechies wavelet of order (5) and six multiresolution levels.
- 4- Coiflets wavelet of order (5) and six multiresolution levels.

For each selected wavelet, the housing permits data was first analyzed for six multiresolution levels and de-noised using the Fixed Form threshold. After de-noising, the series were modeled using Box-Jenkins method and forecasting criteria were computed and compared with those in the first method mentioned before. Table (1) shows the performance measures for ARMA (3, 2) model of the original and de-noised data using the Fixed Form thresholding (Data are summarized in appendix). It is clear from the table that when the ARMA model was used, wavelet de-noising reduces forecasting errors depending on forecasting criteria. The reduction is in highest level when the housing permits data was Fixed Form thresholded using Discrete Meyer Wavelet (note from the table 1, the best reduction comparing with all other forecasting criteria used here).

**Table 1: RMSE, MAE and AIC for model of the housing permits data with de-noising data.**

Method	Kind	RMSE	MAE	AIC
First method (original data)	ARMA(3, 2)	6.9348	5.0906	4.0231
Second method (de-noised data)	Haar	5.8546	4.0433	3.6845
	Dmey	4.4692	3.0805	3.1444
	Daubechies(5)	5.0210	3.3499	3.3773
	Coiflet(5)	4.7633	3.2577	3.2719

Filtered observations using wavelet shrinkage filters were modeled once again for each wavelet filter, but this time as ARMA(2,2), ARMA(2,1) and ARMA(2,0) when rebuilding the model instead of ARMA(3,2) after careful modeling and without effect on the suitable and accuracy of the model, then compared with the original model ARMA(3,2).

**Table (2) shows the performance measures for ARMA(3, 2) model of the original and de-noised data when rebuilding ARMA(2,2), ARMA(2,1) and ARMA(2,0) model**

Method	Kind	RMSE	MAE	AIC
First method (original data)	ARMA(3, 2)	6.9348	5.09064	4.0231
Second method (de-noised data) ARMA(2, 2)	Haar	5.8431	4.1295	3.6555
	Dmey	4.4782	3.1075	3.1235
	Daubechies(5)	4.9691	3.3168	3.3315
	Coiflet(5)	4.7741	3.3248	3.2514
Second method (de-noised data) ARMA(2, 1)	Haar	6.2125	4.5223	3.7240
	Dmey	4.5727	3.1951	3.1402
	Daubechies(5)	5.1479	3.4635	3.3772
	Coiflet(5)	4.8468	3.4046	3.2566
Second method (de-noised data) ARMA(2, 0)	Haar	6.2350	4.7214	3.7353
	Dmey	4.5471	3.1994	3.1040
	Daubechies(5)	5.1282	3.4795	3.3445
	Coiflet(5)	4.8151	3.4055	3.2185

It is clear from the table-2 that when the ARMA model was used, wavelet de-noising reduces forecasting errors depending on forecasting criteria. Once again the reduction is in highest level when the housing permits data was Fixed Form thresholded using Discrete Meyer Wavelet (note from the table 2, the best reduction comparing with all other forecasting criteria used here).

## 5. Conclusions

- 1- More information could be obtained from a series when using Wavelet Shrinkage technique, specially the wavelet filter Discrete Meyer, forecasting errors can be reduced depending on some performance measures.
- 2- The ability of lowering the order of the estimated model of housing permits data for filtered observations using Wavelet Shrinkage as a result of reducing the noise with keeping the efficiency and suitability of that model.

## 6. Recommendations

- 1- Use of the Wavelet Shrinkage technique in reduction of the model order ARMA(p,q) with keeping the efficiency and suitability of that model.
- 2- Conduct a study on the use of Mid Threshold, Firm Threshold, Non-Negative Garrote Threshold in the reducing the noise with keeping the efficiency and suitability of that model.

## References

- [1] Alrumaih M. and Al-Fawzan A. Time series forecasting using wavelet denoising an application to Saudi stocks index, J.King Saudi Univ., 14, 221-234 (2002).
- [2] Alwadi S., Mohd Tahir Ismail, Alkhahazaleh M.H. and Samsul Addul Karim, Selecting Wavelet Transforms Model in Forecasting Financial Time Series Data, Based on ARIMA Model, Applied Mathematical Sciences, 5, 315-326 (2011).
- [3] Bruce Andrew G. and Hong-ye Gao Understanding waveshrink : Variance bias estimation, Biometrica, printed in great Britain, 1996.
- [4] Burrus C.S., Gopinath R.A. and Guo H. Introduction to wavelets and wavelet transforms, Printice - Hall, Inc, Upper Saddle River, N.J., USA, 1998.
- [5] Cascio L.L. Wavelet analysis and denoising: New tools for economists, Queen Marry press, university of London, No.600, 2007, PP.22-26.
- [6] Fugal D. L. Conceptual wavelets in digital signal processing, Space and Signal technical publishing, San Diego, USA, (2014).
- [7] Gencay R., Selcuk F. and Whitcher B. An introduction to wavelets and other filtering methods in finance and economics, Academic press, 2001.
- [8] Makridakis S., Wheelwright S.C. and Hyndman R. J. Forecasting methods and applications, 3<sup>rd</sup> ed., John Wiley and Sons, Inc, NYC, USA, 1998.
- [9] Misiti M., Misiti Y., Oppenheim G. and Poggi J.M. Wavelet toolbox - user's guide, 1st version, the mathworks, Inc, Natick, MA, USA, 1996.





[10] Percival D.B. "An introduction to the wavelet analysis of time series", IEEE, Int'l, Kansas city, Missouri, USA, (2008), PP.36-39.  
 [11] Raimondo M. Wavelet shrinkage via peaks over threshold, a paper submitted to the school of Mathematics and Statistics ,University of Sydney Australia, PP.14,(2002).  
 [12] Tsay R.S. Analysis of financial time series, John Wiley and Sons, Inc, USA, (2002).  
 [13] Yaffee R. Introduction to time series analysis and forecasting with applications of SAS and SPSS, Academic Press , Inc, NYC, USA, (1999).

**Appendix Observations of the original model ARMA (3, 2) and filtered using wavelet shrinkage housing permits**

S	ARMA(3,2)	W H S	W dme S	W db2 S	W db3 S	W db4 S	W db5 S	W coif1 S	W coif2 S	W coif3 S	W coif4 S	W coif5 S
1	83.3	85.1	84.8	82.1	84.0	86.7	84.4	85.0	86.7	86.3	86.6	85.5
2	83.2	85.1	87.4	81.5	88.9	88.5	88.0	86.8	88.0	87.6	88.2	86.9
3	105.3	106.3	104.7	100.3	104.3	103.2	103.4	103.8	104.6	104.3	104.9	103.6
4	117.7	114.5	113.7	114.0	117.0	115.5	115.0	113.2	113.4	113.2	113.6	112.6
5	104.6	104.6	106.4	105.3	105.5	104.9	105.2	104.0	104.8	105.3	105.7	105.2
6	108.8	104.6	105.1	106.8	104.8	106.3	106.8	105.4	105.1	105.6	105.7	105.3
7	93.9	92.7	95.2	95.7	95.3	96.4	96.4	94.9	94.9	95.8	96.1	95.9
8	86.1	89.0	87.3	87.9	85.1	86.2	87.2	90.3	88.9	88.2	89.0	88.2
9	83.0	86.0	87.0	86.7	87.5	87.9	85.2	84.0	86.4	86.2	88.5	87.3
10	102.4	101.2	100.3	101.3	104.1	104.1	99.5	102.6	101.5	100.0	103.1	101.0
11	119.6	119.6	118.7	119.9	118.8	120.4	118.3	118.9	120.9	118.7	121.9	119.5
12	141.4	137.2	137.2	137.4	140.8	140.8	137.7	136.0	139.4	137.9	140.6	138.6
13	158.6	155.8	150.8	154.1	152.9	154.6	154.3	156.1	154.6	152.5	153.6	152.4
14	161.3	155.8	157.4	161.4	158.8	158.9	159.5	154.9	157.6	157.9	158.3	158.3
15	158.2	154.9	153.0	152.4	151.6	154.6	153.9	155.0	153.8	153.6	152.7	153.5
16	136.1	137.0	137.7	133.5	134.7	137.5	133.9	136.7	137.4	138.1	136.8	137.9
17	121.9	120.3	119.6	117.5	120.1	120.5	115.5	120.8	120.5	121.3	119.7	120.6
18	97.7	100.3	101.1	99.1	100.4	100.7	99.1	99.5	100.6	102.2	100.7	101.5
19	103.3	104.6	100.5	102.0	102.5	101.8	103.0	99.5	101.1	102.7	101.1	101.6
20	92.7	98.2	92.5	97.1	95.9	94.7	97.1	94.5	93.9	94.5	93.2	93.6
21	106.8	106.0	101.0	105.2	104.1	105.0	106.7	105.8	102.7	102.3	101.6	102.0
22	102.1	105.5	104.7	103.8	101.1	106.6	105.3	104.2	104.0	104.2	104.1	104.8
23	110.3	110.6	111.9	108.6	106.6	111.2	107.9	108.3	110.8	111.3	111.4	112.3
24	114.1	110.6	113.5	111.7	112.0	110.9	110.1	109.2	110.3	111.0	111.4	112.7
25	109.1	106.2	110.1	107.5	109.6	108.2	108.7	107.7	108.1	108.1	108.1	109.7
26	105.4	106.2	104.1	105.2	104.8	101.2	103.3	104.5	103.5	103.4	103.0	104.9
27	97.6	101.1	100.2	101.4	100.7	98.8	99.7	100.2	99.2	99.5	99.3	101.4
28	100.7	101.1	101.2	101.5	98.9	101.0	102.1	103.2	100.9	100.6	100.7	102.6
29	102.7	104.6	105.4	106.1	104.0	104.6	105.9	104.6	105.0	105.0	105.8	107.3
30	110.9	108.7	110.9	109.4	113.2	111.6	110.9	111.9	111.4	110.6	111.5	112.2
31	120.2	119.2	118.8	119.7	120.6	120.3	118.8	118.6	118.5	117.6	118.6	118.9
32	131.3	126.1	128.2	128.1	130.0	130.7	129.1	125.7	127.4	127.4	128.3	128.7
33	138.8	133.7	132.6	134.8	132.4	133.3	134.4	135.6	134.0	133.8	133.8	134.3
34	130.9	130.0	128.3	131.8	126.8	126.9	128.9	126.8	127.8	128.8	128.4	129.4





35	123.1	120.9	120.0	120.5	120.7	120.7	121.3	119.7	118.7	120.1	119.2	120.5
36	110.8	112.8	114.1	111.4	109.8	112.2	114.2	113.0	112.9	114.9	113.9	115.3
37	108.8	105.8	110.7	106.9	107.1	106.8	108.2	109.9	109.6	111.6	110.3	111.6
38	103.8	105.0	106.5	101.2	105.7	103.3	102.7	103.4	103.0	105.7	104.8	106.4
39	97.0	97.1	100.1	98.7	99.8	98.4	98.9	96.4	97.1	100.1	99.3	100.5
40	93.2	97.1	93.7	95.3	93.4	92.4	95.1	94.8	93.5	95.2	93.9	94.5

S	ARMA(3,2)	W H S	W dme S	W db2 S	W db3 S	W db4 S	W db5 S	W coif1 S	W coif2 S	W coif3 S	W coif4 S	W coif5 S
41	89.7	91.9	91.4	92.5	91.2	89.9	92.7	92.6	90.9	91.6	90.4	90.7
42	89.9	91.9	93.1	89.5	89.9	90.5	91.4	91.2	90.6	92.2	91.8	92.0
43	90.2	91.9	93.2	92.0	88.8	90.3	90.4	89.5	91.5	93.6	93.7	93.6
44	89.6	91.9	89.8	93.1	88.6	89.9	88.9	89.5	89.0	90.4	90.4	90.1
45	85.8	89.3	90.1	90.1	89.9	90.9	90.3	87.2	88.3	89.7	89.9	89.5
46	96.9	96.2	99.3	96.1	99.8	99.2	98.4	99.6	99.8	100.6	100.6	99.8
47	112.7	113.2	111.7	109.8	110.5	109.1	107.8	111.4	112.1	113.1	113.3	112.5
48	122.7	119.1	118.2	121.4	120.6	118.2	117.2	117.7	117.4	118.0	118.3	117.9
49	119.8	117.3	118.4	117.7	119.1	118.3	119.0	117.8	117.3	118.1	118.2	117.9
50	117.4	117.3	116.6	118.1	112.5	114.8	117.2	114.3	115.0	115.9	115.8	115.6
51	111.9	111.5	113.2	110.3	108.8	110.1	111.1	111.3	110.4	111.2	111.4	111.5
52	104.7	108.5	107.1	104.7	102.8	105.2	104.8	106.0	105.7	106.2	106.7	106.4
53	98.3	97.4	100.1	101.3	99.7	100.3	98.3	100.7	100.9	101.0	102.0	101.1
54	94.9	97.4	95.0	97.3	97.2	96.9	94.9	97.1	96.3	95.6	97.1	95.6
55	93.3	95.9	92.3	95.7	94.3	94.1	93.4	92.3	93.0	92.1	93.7	92.0
56	90.9	95.9	92.2	93.5	91.9	92.6	94.4	94.2	93.4	92.3	93.3	92.2
57	91.9	96.3	95.3	95.0	94.3	94.4	97.1	94.7	95.6	95.0	95.5	95.5
58	97.2	97.4	100.7	95.5	98.9	100.0	100.5	100.1	100.2	99.8	99.5	100.3
59	104.7	105.5	105.1	101.7	103.7	103.1	103.9	105.8	105.2	104.5	103.6	104.7
60	107.7	105.5	106.4	106.4	110.2	104.1	106.0	106.6	106.7	106.2	106.2	106.7
61	108.2	109.7	106.7	108.7	110.9	106.7	108.5	107.6	107.5	106.8	107.9	107.3
62	110.7	109.7	108.3	111.6	109.9	109.8	112.2	109.3	109.6	108.4	110.4	108.5
63	113.2	111.1	110.0	112.6	111.2	111.9	114.3	110.8	111.6	110.3	112.7	110.0
64	114.6	111.1	111.1	114.2	112.1	113.5	113.9	113.1	112.6	111.5	113.3	111.1
65	112.2	114.0	113.8	114.5	113.4	114.7	114.4	115.0	113.7	113.5	114.3	113.6
66	120.2	117.8	119.0	118.4	118.3	117.7	117.5	119.1	117.7	118.3	118.0	118.6
67	122.1	120.9	123.2	120.6	120.9	119.4	120.2	124.1	121.4	122.4	121.1	122.7
68	126.6	121.2	123.1	123.2	124.3	120.7	121.8	122.5	120.5	121.5	120.2	122.2
69	122.3	119.0	120.2	119.2	122.2	119.8	121.7	119.0	118.4	118.7	117.7	119.3
70	115.9	116.8	117.3	117.0	118.0	116.1	118.4	116.6	116.8	116.5	116.1	117.2
71	116.9	116.6	115.3	115.2	114.4	115.2	117.2	113.9	115.4	114.6	114.6	115.2
72	110.1	114.0	113.4	113.4	109.4	113.0	113.3	112.2	112.9	111.7	112.1	112.8
73	110.4	110.8	111.4	111.4	109.1	110.9	110.9	109.9	110.7	109.8	110.4	111.2
74	108.9	110.8	110.5	109.4	110.3	108.1	109.5	110.4	111.1	109.6	109.8	110.6
75	112.1	112.4	112.2	112.8	112.3	109.0	110.3	110.2	111.1	109.9	110.3	111.5
76	117.6	113.7	114.5	114.8	115.7	113.9	114.5	112.7	113.1	112.2	112.3	113.9



77	112.2	110.4	110.7	108.6	106.8	108.2	112.2	110.8	111.9	110.3	109.2	110.9
78	96.0	98.4	97.6	97.0	93.3	91.9	96.0	95.2	96.5	95.7	94.7	97.0
79	78.0	79.2	81.1	82.5	79.5	79.7	80.5	79.3	80.1	79.4	78.0	80.2
80	66.9	72.3	71.3	69.0	66.8	69.0	71.5	72.0	72.1	70.6	69.1	70.3

### پوخته

لهم تووژينه وه يه دا توانرا به راورد نيوان موديله كانى (بوکس - جينکنز) ي راسته هتيلى  $ARMA(p,q)$  خه ملينراو له داتاي زنجيره کاتيبه كان بکريت پيش و پاش به کارهتنيانى فلتهرى کهم کردنه وهى شه پولى بچووک (به کارديت بۆ چاره سه رى گرفتى هه له (پيس بوون) ي داتا گهر هه يت) پاشان توانرا پله ي موديله خه ملينراوه که کهم بکريت وه ( ي نه وهى کاريگه رى له سه ر گونجان و وورديينى موديله خه ملينراوه که بکات ) ، و دووباره به راورد کردنيان له گه ل موديله راسته هتيله كان که خه ملينراوه له داتا بنچينه يه که دا پشت به ست به هه ندئ پيوه رى ئامارى وهک (ره گى دووجاي ناوه ندى دووجاي هه له  $RMSE$  ، ناوه ندى هه له ي رووت  $MAE$ ، پيوه رى نه کايكى بۆ زانبارى  $AIC$  نه ویش به ج به ج کردنيان له سه ر چوار جور لى داتاي زنجيره کاتيبه كان که ده گونجيت له گه ل موديله كانى سه ره وه نه ویش (مووله تى خانووبه را) به به کارهتنيانى به رنامه كانى  $Statgraphics$  ,  $NCSS$  .  $MATLAB$  تووژينه وه که گه يشته نه وهى که وا توانستى فلتهره كانى کهم کردنه وهى شه پولى بچووک بۆ چاره سه رکردنى هه له (پيس بوون) ي داتا وه به ده ست هتنيانى موديلى خه ملينراوى توانست و به تاييه ت فلتهرى  $dmeY$ ) له گه ل پارچه ي پيگه ي نه رم که ئاسته که ي خه ملينراوه به ريگاي شيوازي نه گور  $(Fixed Form)$  و تواناي وه به ده ست هتنيانى موديلى راسته هتيلى پله ي که متر و توانستى زورتر بۆ داتا به فلتهر کراوه كان به به راورد له گه ل موديله خه ملينراوه كان له داتا بنچينه يه که دا.

### ملخص

تم في هذا البحث مقارنة نموذج (بوکس - جينکنز) الخطية  $ARMA(p,q)$  المقدره من بيانات السلاسل الزمنية قبل وبعد ترشيح التقليل المويجي (المستخدم لمعالجة مشكلة التلوث أو الضوضاء إن وجدت في تلك المشاهدات) ومن ثم تخفيض رتبة الأنموذج المقدر من المشاهدات المرشحة (مع الحفاظ على دقة وملائمة الأنموذج المقدر) وإعادة مقارنته مع النماذج الخطية المقدره للمشاهدات الأصلية بالإعتماد على بعض المعايير الإحصائية وتشمل (الجذر التربيعي لمتوسط مربعات الخطأ  $RMSE$ ، متوسط الأخطاء المطلقة  $MAE$  ومعيار أكايكي للمعلومات  $AIC$ ) وذلك من خلال تناول تطبيق عملي لسلسلة زمنية تتفق مع الأنموذج تمثل (الرخص السكنية) باستخدام البرامج الإحصائية  $Statgraphics$  ,  $NCSS$  ,  $MATLAB$  . توصلت الدراسة إلى كفاءة مرشحات التقليل المويجي في معالجة مشكلة الضوضاء والحصول على نماذج مقدره كفاءة وبالتحديد مرشح التقليل المويجي  $dmeY$  مع قطع العتبة الناعمة المقدر مستواها بطريقة الصيغة الثابتة وإمكانية الحصول على نماذج خطية ذات رتب أقل وكفاءة أعلى للمشاهدات المرشحة مقارنة مع ما يقابلها من النماذج المقدره من المشاهدات الأصلية.